

A METHOD OF JOINT DETERMINATION OF THE EFFECTIVE COEFFICIENTS OF
HORIZONTAL MIXING OF LARGE PARTICLES AND OF EXTERNAL HEAT EXCHANGE
IN AN ORGANIZED FLUIDIZED BED

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A method is proposed for the joint determination of the coefficients of horizontal particle diffusion and external heat exchange in a stagnant fluidized bed.

The problem of studying the laws of transfer of heat and dispersed material in fluidized beds of large ($d > 0.6$ mm) particles with a fixed nozzle (usually bundles of tubes) is now urgent, especially in connection with the creation of experimental-industrial boiler plants for burning solid fuel in a fluidized bed. The mixing of particles and heat exchange between the bundle of tubes and the bed occupy an important place in the wide circle of such problems. Along with the purely theoretical problems (especially those characteristic of the description of mixing, not fully solved yet), the study of these questions is also hindered by considerable methodological and experimental difficulties. As a rule, they develop in the determination of the effective coefficients of particle diffusion and the average coefficients of external heat exchange in large apparatus. In this connection, in the literature there are only fragmentary separate data on the values of the effective coefficients of (axial and radial) diffusion of large particles in sufficiently large installations [1-3]. A similar situation is also characteristic of investigations of external heat exchange [4].

From the above it is clear that the problems of the development of methods of measuring the above-mentioned parameters which are simple, sufficiently well-founded theoretically, and suitable for the study of large-scale fluidized beds are important and require their own solutions. The method proposed below for describing heat transfer in an organized fluidized bed of large particles is investigated in just such a formulation in the present report.

Let us consider the nonsteady process of the transfer of preliminarily heated particles in a stagnant fluidized bed of cold particles fluidized by a cold gas. Taking the process of particle mixing as purely diffusional (horizontally this is admissible [5]), the weighted-average (over hot and cold particles) temperature of the solid phase (t_s) and the surface temperature of the nozzle (t_n) can be found from the solution of the system of equations

$$\frac{\partial t_s}{\partial \tau} = D \frac{\partial^2 t_s}{\partial y^2} - \frac{\{C\rho\} u (t_s - t_0)}{H \varepsilon_n} - \frac{\alpha s_n (t_s - t_n)}{\rho_{be} C_s \varepsilon_n}, \quad (1)$$

$$\frac{\partial t_n}{\partial \tau} = \frac{\alpha s_n (t_c - t_n)}{\rho_n C_n (1 - \varepsilon_n)}$$

with the appropriate boundary conditions reflecting the initial distributions of t_s and t_n and the heat exchange between the bed and the walls of the apparatus:

$$t_s(y, 0) = t_n(y, 0) = \begin{cases} t_c, & 0 \leq y < Rl, \\ t_0, & Rl < y \leq l, \end{cases} \quad (2)$$

$$D \frac{\partial t_s}{\partial y} = \alpha^* (t_s - t_w) / \rho_{be} C_s \varepsilon_n, \quad y = 0,$$

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with t_w being a variable in general. In the first equation of the system (1) it was assumed that the gas temperature at the exit from the bed is equal to the particle temperature. This is true at $H \geq H_\alpha$, where H_α is the height of the active section (interphase heat exchange under the conditions of the balanced problem [6]). In writing the second equation of (1) it was assumed that the heating (or cooling) of the nozzle occurs in a zero-gradient mode. This is valid for $Bi < 0.4$ [6].

Because the system (1) with the conditions (2) is rather complex, let us try to simplify the problem. We designate $S_0 = \{C_p\}u(t_s - t_0)/He_n + \alpha_{sn}(t_s - t_n)/\rho_b e C_{sn}$. As is done in [7], we replace the heat exchange between the bed and the walls of the apparatus by the action of an equivalent volumetric heat sink with a power S_1 . With allowance for this, the first equation of (1) will be

$$\frac{\partial t_s}{\partial \tau} = D \frac{\partial^2 t_s}{\partial y^2} - (S_0 + S_1). \quad (3)$$

We expand the function $S = S_0 + S_1$ in a Taylor series by powers of the relative dimensionless particle temperature $\theta = (t_s - t_0)/(t_c - t_0)$,

$$S/(t_c - t_0) = \beta_0 + \beta_1 \theta + \beta_2 \theta^2 + \dots \quad (4)$$

Since $\theta \sim R$ at $y/l > R$ [8], while $R = 0.05-0.2$ in experiments on measurement by the method of a plane heat pulse (used in the present report), as a rule, it is admissible to leave only the term linear with respect to θ in the expansion (4) (from the physical meaning of the problem it is clear that $\beta_0 = 0$).

With allowance for the simplifying assumptions made, we represent Eq. (3) for determining the dimensionless excess temperature of the bed in the form

$$\frac{\partial \theta}{\partial Fo} = \frac{\partial^2 \theta}{\partial \xi^2} - Pe \theta. \quad (5)$$

The corresponding boundary conditions are

$$\theta(\xi, 0) = \begin{cases} 1, & 0 \leq \xi < R, \\ 0, & R < \xi \leq 1, \end{cases} \quad (6)$$

$$\partial \theta(0, Fo)/\partial \xi = \partial \theta(1, Fo)/\partial \xi = 0.$$

With the condition $Pe = \text{const}$ * the solution (5)-(6) has the form [8]

$$\theta = \exp(-PeFo) \left[R + 2 \sum_{n=1}^{\infty} \frac{\sin n\pi R}{n\pi} \cos n\pi \xi \exp(-n^2 \pi^2 Fo) \right]. \quad (7)$$

Thus, the expansion (4) simplifies the system (1), while its second equation allows one to determine t_n using (7).

The factors $e^{-n^2 \pi^2 Fo}$ in the series (7) decrease very rapidly with an increase in Fo . For $Fo > 0.6$ the expression of the type (7) for θ is simplified,

$$\theta \cong R \exp(-PeFo) = R \exp(-\beta_1 \tau), \quad (8)$$

i.e., a regular mode [7] of cooling of the bed begins, with a temperature gradient being absent from the system, as seen from (8).

Special tests were conducted to verify the correctness of the assumptions made, permitting the development of a sufficiently simple joint method of determining the coefficients D and α (see below).

The experimental installation consists of a column with a rectangular cross section of 400×250 mm and a height of 1.5 m. For visual observation of the bed the front wall of the apparatus was made of plastic. The gas-distribution grid consisted of two perforated plates (openings 10 mm in diameter with a spacing of 14 mm, arranged in checkerboard order), between

*The presumed independence of Pe from ξ , Fo , and θ may be justified by comparing the results obtained with experimental data.

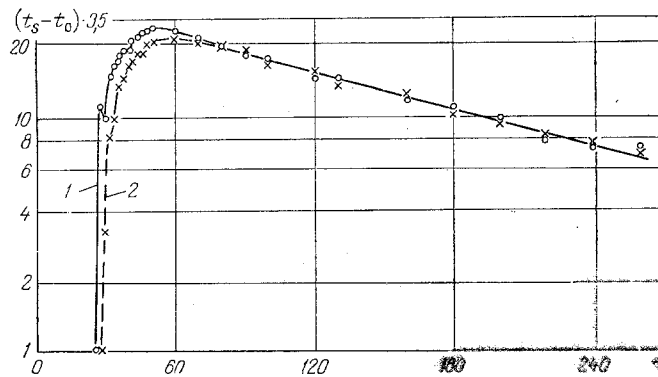


Fig. 1. Time variation of excess temperature of bed at the points $\xi = 0.49$ (1) and $\xi = 0.87$ (2); $u = 87$ cm/sec. $(t_s - t_0) \cdot 0.5$, °C; τ , sec.

which a layer of closely woven cloth was pressed. The hydraulic resistance of the grid in the range of filtration velocities of 0.50-1.13 m/sec is described by the relation $\Delta p = 4 \cdot 10^3 u^{1.35}$, Pa. A packet of thin-walled ($\delta = 1.5$ mm), horizontal, Duralumin tubes with an outer diameter of 30 mm was installed in the fluidized bed in a checkerboard arrangement ($S_h \times S_v = 60 \times 60$ mm, $\epsilon_n = 0.8$).

As the dispersed material we used quartz sand with $d = 0.6$ mm ($u_0 = 20$ cm/sec; $\rho_s = 2540$ kg/m³). In the experiments the height of the stationary bed was maintained in the range of 24-25.5 cm. In this case the bed did not go beyond the limits of the bundle of tubes even at the maximum air filtration velocities. The flow rate of the fluidizing agent was measured with a diaphragm with an error of no more than 3%.

A movable barrier was mounted at a distance of 60 mm from the left end of the apparatus, with the help of which the initial condition in (6) was achieved in the tests ($R = l_0/l = 0.15$).

The procedure for conducting the experiments consisted in the following. The bed was fluidized by air at room temperature with an assigned filtration velocity. The movable barrier was inserted into the apparatus, dividing the system into two parts: a heating chamber ($0 \leq \xi < R$) and a working chamber ($R < \xi \leq 1$). A small portion (about 1% of the volume of the bed) of preheated material was poured into the heating chamber. After a certain time, required for the mixing of the hot particles in the heating chamber (about 5-10 sec), the movable barrier was rapidly removed (in a time of not more than 0.5 sec) from the bed and the values of t_s and t_n at different points of the bed were measured with time by a system of thermocouples. In the tests the values of t_c and t_0 were about 60-70 and 15°C, respectively. The tube for sensing the temperature t_n was mounted in the bed at a distance of 220 mm from the grid and 80 mm from the right end of the apparatus and was thermally insulated from the other tubes. The thermocouples measuring t_s were movable and could be placed at different points of the apparatus.

The characteristic form of the function $t_s(\tau) - t_0$ for different points of the bed is presented in Fig. 1 in semilogarithmic coordinates. It is well seen that, starting at about $\tau = 80$ sec, a regular mode of zero-gradient cooling of the bed begins in complete accordance with (8). This confirms the correctness of the simplifying assumptions made.

As shown in [8], at $Pe = 0.05-4.00$ (these values of Pe occur in practice, as a rule), when the variation in the temperature t_s at the points $\xi = 1$ and 0.5 is fixed for times Fo close to Fo^{\max} (corresponding to the maximum values of t_s), one can be confined to the first and to the first three terms in the series (7), respectively. With allowance for this, from the condition of onset of the temperature maximum $d\theta/dFo = 0$ we get [8]

$$Fo^{\max} = \frac{D\tau_{\max}}{l^2} = \Phi_n(Pe, R), \quad (9)$$

$$\Phi_n = \frac{1}{(n\pi)^2} \ln \left[\frac{2(n^2\pi^2 + Pe) \sin n\pi R}{n\pi R Pe} \right]$$

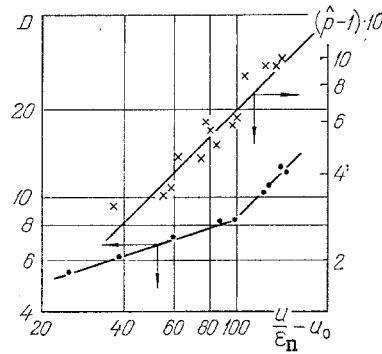


Fig. 2. Dependence of coefficient of horizontal diffusion and relative expansion $\hat{p} - 1$ of the bed on the excess air velocity. $u/\epsilon_n - u_0$, cm/sec; D , cm²/sec.

and $n = 1$ and 2 correspond to the points $\xi = 1$ and 0.5 . The form of the function ϕ_n is presented in [8]. Calculations of D are made from (9) by the method of successive approximations [8]. Here the coefficient β_1 , appearing in Pe is determined in the usual way in accordance with (8) from the slope of the straight section of the experimental function in Fig. 1. Thus, the proposed method of determining D allows for the total influence of the nozzle, the walls of the apparatus, and the filtering gas and, naturally, it permits an increase in the accuracy in determining D in tests with thermal marking of particles, in which only the heat sink with the departing gas was allowed for earlier [8].

The equation for the error in determining D in accordance with (9) (taking $(\sin n\pi R)/n\pi R \approx 1$ and denoting $2n^2\pi^2/Pe$ as γ) has the form

$$\frac{\delta D}{D} = \left\{ (2 + \gamma) \ln(2 + \gamma) / [(2 + \gamma) \ln(2 + \gamma) - \gamma] \right\} \frac{\delta \tau}{\tau} + \left\{ [(2 + \gamma) \ln(2 + \gamma) + \gamma] / [(2 + \gamma) \ln(2 + \gamma) - \gamma] \right\} \frac{2\delta l}{l} + \left\{ \gamma / [(2 + \gamma) \ln(2 + \gamma) - \gamma] \right\} \frac{\delta \beta_1}{\beta_1}. \quad (10)$$

For a specific mode ($u = 64$ cm/sec; $\tau_{\max} = 60$ sec; $\beta_1 = 0.0039$ 1/sec; $D = 8.7$ cm²/sec; $\xi = 1$) we obtain from (10)

$$\frac{\delta D}{D} = 1.37 \frac{\delta \tau}{\tau} + 3.52 \frac{\delta l}{l} + 0.38 \frac{\delta \beta_1}{\beta_1} \cong 0.18,$$

taking $\delta \tau / \tau = 0.1$, $\delta l / l = 0.01$, and $\delta \beta_1 / \beta_1 = 0.03$. If we only allow for the heat sink with the filtering air, as in [8], then $\delta \beta_1 = 0.002$ 1/sec and $0.38 \delta \beta_1 / \beta_1 = 19\%$. Thus, in this specific case the accuracy in determining D rises to 19%.

The data obtained on the measurement of D as a function of the velocity $u/\epsilon_n - u_0$ are shown in Fig. 2. The experimental values of the expansion of the bed, obtained by visual observations of the bed through the transparent front wall, are presented in the same figure.

Knowing the experimental functions $t_s(\tau)$ and $t_n(\tau)$ measured at nearby points, the second equation of (1) allows one to determine the coefficient of external heat exchange from the equation

$$\alpha = \frac{mC_n \frac{\partial t_n}{\partial \tau}}{\pi d_{\tau} L (t_s - t_n)}, \quad (11)$$

where m is the mass of the thermally insulated tube. The Biot number for the tube is on the order of $2 \cdot 10^{-3}$, which lets one take its heating (and cooling) as zero-gradient.

The form of the $t_s - t_0$ and $t_n - t_0$ curves obtained at the position of the thermally insulated tube is shown in Fig. 3 in semilogarithmic coordinates.

The values of α calculated from (11) using a computer from the experimental curves of $t_s(\tau)$ and $t_n(\tau)$ are presented in Fig. 4. As is seen, a pronounced decrease (up to 30%) in

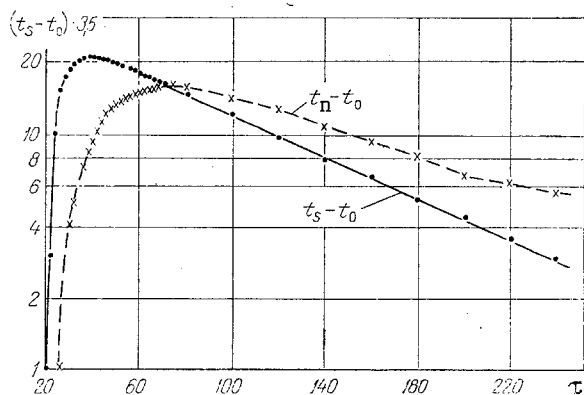


Fig. 3. Time variation of excess temperature of the bed and the thermally insulated tube. $u = 120$ cm/sec; $D = 11.6$ cm²/sec; $\alpha = 150$ W/(m²·°K).

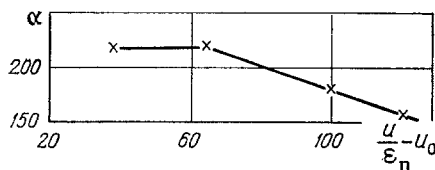


Fig. 4. Dependence of the coefficient of heat exchange α on the excess air velocity $u/\epsilon_n - u_0$.

the heat-exchange intensity is observed at $u_0/\epsilon_n - u_0 > 60$ cm/sec. This is evidently caused by the considerable decrease in the concentration of solid particles at increased air filtration velocities (see Fig. 2). The instrumental error of the method of determining α [9] is

$$\frac{\delta\alpha}{\alpha} = \frac{\delta m}{m} + \frac{\delta L}{L} + \frac{\delta d_t}{d_t} + \sqrt{\frac{W^2 + (\delta\alpha_i^0/\alpha_i^0)^2}{N-1}}, \quad (12)$$

where W is the coefficient of variation of the values of α_i^0 relative to α^0 , while $\delta\alpha_i^0/\alpha_i^0$ is determined by the expression

$$\frac{\delta\alpha_i^0}{\alpha_i^0} = \frac{\delta t}{(t_s - t_n)_i} + 0.5 \ln \left[1 + \frac{\delta t}{(t_n - t_0)_i} \right]. \quad (13)$$

Using the values of $W = 0.3$ and $\delta\alpha_i^0/\alpha_i^0 = 0.07$, from (12)-(13) we obtain $\delta\alpha/\alpha = 6\%$ (for $N = 30$).

Thus, the proposed method is a joint one: It allows one to determine from one experiment the coefficient of horizontal particle diffusion and the coefficient of external heat exchange between the bed and the bundle of tubes. It allows for the influence of the tubes and the walls of the apparatus on the quantity D being measured. The special features of the thermal marking of particles (the need to separate the tracer after each test drops out) permit the use of this method to study large-scale apparatus containing fluidized beds.

In conclusion, we note that the determination of D by the proposed method is possible only for situations when the temperature maximum of the curve $t_s(\tau)|_{y=\text{const}}$ is sufficiently clearly expressed; only then can τ_{max} be reliably fixed. As a rule, this corresponds to the case of sufficiently large particles ($d \geq 0.6$ mm) and modes with not very low filtration velocities.

NOTATION

C_f, C_s, C_n , specific heat capacities of gas, particles, and nozzle material, respectively, at constant pressure; D , effective coefficient of particle diffusion horizontally (coefficient of horizontal thermal diffusivity of the bed); d , equivalent particle diameter; d_t , tube

diameter; H_0 , H , heights of bed at gas filtration velocities u_0 and u , respectively; H_a , height of active section; l , width of bed; L , tube length; l_0 , width of heating chamber; N , number of partition intervals; $\hat{p} = H/H_0$, expansion of bed; $R = l_0/l$; s_n , surface area of nozzle per unit volume of bed; S_H , S_V , horizontal and vertical spacings between tubes; t_c , t_0 , t_s , t_n , t_w , initial temperature of heating chamber, entrance temperature of gas, particle temperature, nozzle temperature, and temperature of apparatus walls, respectively; u_0 , u , velocity of start of fluidization and gas filtration velocity; y , horizontal coordinate; α^* , α , coefficient of external heat exchange between bed and walls of apparatus and nozzle; $\alpha^0 = (\partial t_n / \partial \tau) / (t_s - t_n)$; β_0 , β_1 , β_2 , ..., coefficients in (4); δ , thickness of tube wall; ϵ_b , bubble concentration in bed; ϵ_0 , porosity of emulsion phase of bed; ϵ_n , porosity of nozzle; $\theta = (t_s - t_0) / (t_c - t_0)$, dimensionless relative temperature of particles; λ_n , coefficient of thermal conductivity of nozzle material; $\xi = y/l$; ρ_f , ρ_s , ρ_n , densities of gas, particles, and nozzle material, respectively; $\rho_{De} = \rho_s(1 - \epsilon_0)(1 - \epsilon_b)$, average density of bed; τ , time; τ_{max} , time of onset of temperature maximum at a selected point of the bed; $\{C_p\} = C_{pf}/C_{sp}e$; $Fo = D\tau/l^2$, Fourier number; $Pe = \beta_1 l^2/D$, Péclet number; $Bi = \alpha\delta/\lambda_n$, Biot number.

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